Mechanical Vibration

Single Degree of Freedom UNDAMPED SYSTEM

Forced Disk With Nonlinear Spring Report with 1DOF

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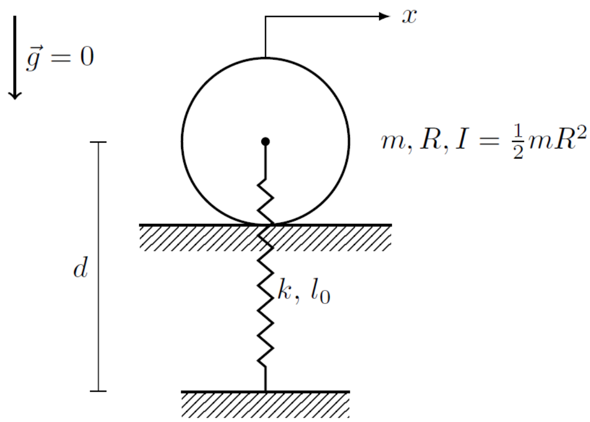
Scheme of the system

Scheme of the real object is presented in the figure. It was obtained by the analysis of real mechanism.



Analysis of the scheme allows to claim its number of degrees of freedom which is 1

Kinetic energy

Kinetic energy of the system has a following form:

$$T = \frac{3m_1\dot{x}^2}{4} \tag{1}$$

Determined formula specify energy of the system related to its inertial properties.

Potential energy

Potential energy of the system has a following form:

$$V = \frac{k\left(l_0 - \sqrt{d^2 + x^2}\right)^2}{2} \tag{2}$$

The presented relationship describes the interaction of potential force fields in which the object is located.

Lagrangian of the system (Lagranges function)

System Lagrangian is described by the formula (3):

$$L = -\frac{d^2k}{2} - \frac{kl_0^2}{2} - \frac{kx^2}{2} + \frac{3m_1\dot{x}^2}{4} + kl_0\sqrt{d^2 + x^2} \tag{3}$$

The Euler-Lagrange equations for the case under consideration are as follows:

$$\frac{\mathrm{d}}{\mathrm{dt}}\frac{\partial L}{\partial \dot{x}} + \frac{\partial D}{\partial \dot{x}} - \frac{\partial L}{\partial x} = Q_x^N \tag{4}$$

Subsequent derivatives obtained with the Euler-Lagrange equations are as follows:

$$\frac{\partial L}{\partial x} = -kx + \frac{kl_0x}{\sqrt{d^2 + x^2}} \tag{5}$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{3m_1\dot{x}}{2} \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{x}} = \frac{3m_1\ddot{x}}{2} \tag{7}$$

$$\frac{\partial D}{\partial \dot{x}} = 0 \tag{8}$$

The results of the presented operations are used to determine the equations of motion of the system.

Equation of motion

Using the calculated derivatives, the equation of motion is based on the appropriate formula. System equation of motion is described by the formula (9)

$$kx - F\cos(\Omega t) + \frac{3m_1\ddot{x}}{2} - \frac{kl_0x}{\sqrt{d^2 + x^2}} = 0 \tag{9}$$

The determined equations constitute a mathematical dynamic description of the properties of the system. Further analysis allows for an effective analysis of the modeled object's operation and determination of its mechanical parameters.

Linearization of equation of motion

Linearization of governing equations is about finding Taylor series with respect to generalized coordinates, velocities and accelerations in the neighbourhood of the equilibrium point. Following symbols have been introduced to make a simplification.

$$x = x(t) \tag{10}$$

$$\dot{x} = \frac{d}{dt}x(t) \tag{11}$$

$$\ddot{x} = \frac{d^2}{dt^2}x(t) \tag{12}$$

Equilibrium points of the system have following forms:

$$x = 0 \tag{13}$$

$$\dot{x} = 0 \tag{14}$$

$$\ddot{x} = 0 \tag{15}$$

Equation of motion for coordinate $x(t)$ can be presented as:Proper computings requires finding derivatives of generalized coordinates, which are components of Lagrange's equations

$$\frac{d}{dt}x(t)\ \frac{d}{d\frac{d}{dt}x(t)}RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} + \frac{d^2}{dt^2}x(t)\ \frac{d}{d\frac{d^2}{dt^2}x(t)}RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} + x(t)\ \frac{d}{dx(t)}RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} + RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} = 0 \tag{16}$$

The calculated derivatives have a following form:

$$RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} = -F\cos(\Omega t) \tag{17}$$

$$\frac{d}{dx(t)}RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} = -\frac{k(-d+l_0)}{d} \tag{18}$$

$$\frac{d}{d\frac{d}{dt}x(t)}RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} = 0 \tag{19}$$

$$\frac{d}{d\frac{d^2}{dt^2}x(t)}RR_x(t)\Big|_{\substack{x(t)=0\\ \frac{d}{dt}x(t)=0\\ \frac{d^2}{dt^2}x(t)=0}} = \frac{3m_1}{2} \tag{20}$$

The following equation (linearized) can be obtained after substitution of calculated derivatives.

$$kx(t) - F\cos(\Omega t) + \frac{3m_1\frac{d^2}{dt^2}x(t)}{2} - \frac{kl_0x(t)}{d} = 0 \tag{21}$$

Determining fundemental matrix

The matrix of masses and stiffnesses of the system was determined from the equations of motion:

$$M = \left[\frac{3m_1}{2}\right] \tag{22}$$

$$K = \left[-\frac{k(-d+l_0)}{d}\right] \tag{23}$$

The fundamental matrix, on the basis of which the characteristic equation of the considered system Δ was determined, is as follows:

$$A = \left[-\frac{3m_1\omega^2}{2} - \frac{k(-d+l_0)}{d}\right] \tag{24}$$

$$\Delta = k - \frac{3m_1\omega^2}{2} - \frac{kl_0}{d} \tag{25}$$

The fundamental matrix allows you to define a fixed solution. On the other hand, based on the characteristic equation, the eigenfrequencies of the system are determined.

General solution

General solution is presented by expression:

$$X_{g-x(t)} = C_1 e^{-\frac{\sqrt{6}\sqrt{k}t\sqrt{-d+l_0}}{3\sqrt{d}\sqrt{m_1}}} + C_2 e^{\frac{\sqrt{6}\sqrt{k}t\sqrt{-d+l_0}}{3\sqrt{d}\sqrt{m_1}}} \quad (26)$$

General solution describes motion of the analised system - presents displacement i function of time - and is given by considerations about free vibrations of the system

Steady solution

The steady solution is given by the formula:

$$X_{s-x(t)} = \frac{0.667F\cos(\Omega t)}{m_1\left(-\Omega^2 + \frac{0.667k(d-l_0)}{dm_1}\right)} \quad (27)$$

The specific solution is related to the presence of quantities that force motion (vibrations) of the analyzed system.